Summer school on Topology of Manifolds
22nd-26th August 2010
Jagiellonian University
Kracków, Poland

An introduction to surgery with applications to the homeomorphism classification of 4-manifolds

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1 Planned Program

- Monday

  1. Some problems e.g. Kummer surfaces blown up once and connected sum of $\mathbb{C}P^2$ and $S^2 \times S^2$. When are they homotopy equivalent or homeomorphic or dffeomorphic? The Whitehead-Milnor classification of 1-connected 4-dimensional Poincare complexes [W, M1].
  2. The oriented cobordism group: Definitions and the signature as a basic invariant. See:
     http://www.manifoldatlas.him.uni-bonn.de/index.php/Bordism
  3. Characteristic classes (Stiefel-Whitney, Chern and Pontryagin), characteristic numbers and the rational oriented bordism groups [M-S][§4, 14, 15, 19]. The signature theorem in dimension 4. Spin structures and Spin bordism groups in low dimensions [L-M][II, §1, 2, p 92].

- Tuesday

  4. The h-cobordism theorem: statement and idea of proof. (The classic depth reference is [M2]).
  5. The idea of surgery classification of manifolds: Modify a bordism until it is an h-cobordism. A similar problem in homotopy: construction of Eilenberg-Mac Lane complexes and its solution.

6. The definition of surgery on a framed embedded sphere and it's effect in homotopy and homology below the middle dimension [M2][§1, 2, 3].

- Wednesday

7. Control spaces: a) normal maps; b) normal homotopy type; c) the normal k-type. See:
   http://www.manifoldatlas.him.uni-bonn.de/index.php/B-Bordism
   Computation of normal 1 and 2 types of smooth manifolds [T][Part 1: 2] Applications to positive scalar curvature (report) [K][Thm. 1, Cor. 2].
8. Surgery below the middle dimension (in particular: stable normal bundles of dim > dim(base) equal to unstable normal bundles); surgery compatible with the normal structure below the middle dimension is possible [M2][§4] [K][§3].

∗ Afternoon free

- Thursday

9. The stable classification of even-dimensional manifolds with idea of the proof [K][§4].
10. The stable classification of closed oriented 4-manifolds with universal covering non-Spin.
11. The definition of l_5-monoids and construction of the surgery obstruction and indication of the proof that if there is a bit stability the obstruction is elementary [K][§6, Theorem 4].

- Friday

12. Using the normal 3/2 type of a 1-connected 4-manifold show that the obstruction in $l_5(\mathbb{Z}[\pi])$ is in $L_5(\mathbb{Z}[\pi])$ if the intersection forms of the two boundary components agree. Using this and the vanishing of $L_5(e)$ classify all 1-connected 4-manifolds up to h-cobordism [K][§7, Theorem 5].
13. Explaining how these results extend to appropriate non-simply connected 4-manifolds, applications to group actions on 4-manifolds, e.g. involution on the Kummer surface blown up and on $S^2 \times S^2$ blown up are topologically equivalent (but not smoothly). See in part:
    http://en.wikipedia.org/wiki/K3_surface

References

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[L-M] H. B. Lawson and M. Michelsohn, *Spin geometry*, Princeton University Press, Princeton, NJ, 1989.

[M1] J. Milnor, *On simply connected* 4*-manifolds*, 1958 Symposium internacional de topologa algebraica International symposi um on algebraic topology, Universidad Nacional Autnoma de Mxico and UNESCO, Mexico City, (1958) 122–128.

[M2] J. Milnor, *A procedure for killing homotopy groups of differentiable manifolds*, (1961) 3955.

[M3] J. Milnor *Lectures on the h-cobordism theorem*, Notes by L. Siebenmann and J. Sondow. Princeton University Press, Princeton, N.J. 1965

[M-S] J. Milnor and Stasheff, *Characteristic classes*, Ann. of Math. Studies, No. 76. Princeton University Press, Princeton, N. J., University of Tokyo Press, Tokyo, (1974).

[T] P. Teichner, *Topological 4-manifolds with finite fundamental group*, PhD Thesis, University of Mainz, Germany, Shaker Verlag 1992, ISBN 3-86111-182-9. Available in part at:
http://math.berkeley.edu/ teichner/Papers/phd.pdf

[W] J. H. C. Whitehead, *On simply connected,* 4*-dimensional polyhedra*, Comment. Math. Helv. 22, (1949) 48–92.