# Summer School on the Topology of Manifolds

## Budapest, August 15-19, 2011

## Program

We shall begin each day with a series of three 45 minute lectures which will introduce the main ideas for that day. After lunch there will be an extended exercise session and the opportunity for further less formal lectures to elaborate and clarify the main points. Note that Wednesday departs from this model and that there is an extra afternoon lecture planned on Thursday. Unless otherwise indicated, the lectures will be given by Matthias Kreck.

- Monday
  - 1 Overview: main problems and the statement of the $s$-cobordism theorem
  - 2 Normal structures: the normal $k$-type and normal bordism groups [K, §2] [MAP]
  - 3 Surgery below the middle dimension [M, §1-3] [K, §3]
  - E1 First exercise session
  - 4 Informal lecture
- Tuesday
  - 1 The stable classification of $2q$-manifolds [K, §2]
  - 2 Applications: the stable classification of 4-manifolds, bordism of automorphism and stably unique smooth structures
    Goal: The statement of Freedman's classification of closed simply connected topological 4-manifolds [F, Theorem 1.5]
  - 3 The main theorem for odd-dimensional bordisms: the obstruction monoid $l_{2q+1}(\pi)$ and the group $L_{2q+1}(\pi)$ [K, §6]
  - E2 Second exercise session
  - 4 Informal lecture
- Wednesday
  - (a) *Lectures by Zoltán Szabó and András Stipsicz about smooth 4-manifolds*:
    We review the rational blow-down procedure and Luttinger surgery, apply them to construct exotic smooth structures on 4-manifolds with small Euler characteristics, and discuss their relation to symplectic topology and smoothings of surface singularities.
  - ∗ Afternoon excursion

- Thursday

  1 The main theorem for even-dimensional bordisms: the obstruction monoid $l_{2q}(\pi)$ and the group $L_{2q}(\pi)$ [K, §5]

  2 The computation of $L_{2q+1}(e)$ [K-M, §6] and a report on some results for $L_{2q+1}(\pi)$

  3 The h-cobordism classification of closed simply connected 4-manifolds

  E3 Third exercise session

  4 (Crowley) Cancellation of $S^q \times S^q$ and $l_{2q+1}(\pi)$ [C-S]

- Friday

  1 (Crowley) A short review of the classical surgery exact sequence [W, Ch 10]

  2 On the classification of complete intersections [K, §8]

  3 On the classification of non-simply connected 4-manifolds [H-K-T]

  E4 Fourth exercise session

# References


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[F] M. H. Freedman, *The topology of four-dimensional manifolds*, J. Differential. Geom. **17** (1982), no.3, 357–453.

[H-K-T] I. Hambleton, M. Kreck and P. Teichner, *Topological 4-manifolds with geometrically two-dimensional fundamental groups*, J. Topol. Anal. **1** (2009), no. 2, 123–151.

[K-M] M. A. Kervaire and J. W. Milnor, *Groups of homotopy spheres I*, Ann. of Math. **77** (1963), 504-537.

[K] M. Kreck, *Surgery and Duality*, Ann. of Math. **149** no.3 (1999), 707-754.

[MAP] Manifold Atlas Project, *B-Bordism*, http://www.map.him.uni-bonn.de/B-Bordism

[M] J. Milnor, *A procedure for killing homotopy groups of differentiable manifolds*, Proc. Sympos. Pure Math., Vol. III (1961), 39–55.

[P] T. Panov, *Bordism*, Bull. Man. Atl. (2011), 23–29. http://www.boma.him.uni-bonn.de/articles/31

[W] C. T. C. Wall, *Surgery on compact manifolds*, Second edition. Edited and with a foreword by A. A. Ranicki. Mathematical Surveys and Monographs, **69**. American Mathematical Society, Providence, RI, 1999.