The topology of manifolds
Budapest August 2011
Problem sets

1. If you attended the summer school and wish to submit a .tex solution, please email Diarmuid Crowley with the day and number of the exercise(s).
2. Send an email with a .tex file to Diarmuid and the following title:

   BudapestDayEx#

   Make sure that **day** and **exercise number** and the **name of all participants** for the solution is at the top of .tex file.
3. Note that solutions to the problems are due by **Friday September 16th.**
4. Key: :) indicates that a solution has been submitted.
5. See the end of Section 5 for how to claim exercises for the lists of problem solvers.
6. `diarmuidc23@gmail.com`
7. http://www.dcrowley.net/teaching

1 Monday

1. (a) Find a diffeomorphism $\mathbb{C}P^2 \sharp (-\mathbb{C}P^2) \cong S^2 \tilde{\times} S^2$ where the latter manifold is the total space of the non-trival linear 2-sphere bundle over S^2 (recall $\pi_1(SO(3)) \cong \mathbb{Z}/2$).

   (b) Show that the manifolds $S^2 \times S^2$ and $\mathbb{C}P^2 \sharp (-\mathbb{C}P^2)$ have isomorphic homotopy groups π_i for all i but that they are not homotopy equivalent.

2. (a) Let t generate the cyclic group $\mathbb{Z}/5$. Verify that $1 - t - t^4 \in \mathbb{Z}[\mathbb{Z}/5]$ is a unit.

   (b) Define a unital ring homomorphism $\phi \colon \mathbb{Z}[\mathbb{Z}/5] \to \mathbb{C}$ such that $\phi(1 - t - t^4) \notin S^1 \subset \mathbb{C}$.

   (c) Deduce that the Whitehead group $\mathrm{Wh}(\mathbb{Z}/5)$ is infinite.

3. (a) Show that a manifold M is orientable if and only if the normal 0-type of M, $B^0(M)$, is the canonical fibration $BSO \to BO$.

   (b) Show that a manifold M is spinable if and only if the normal 1-type of M, $B^1(M)$, is given by

   $$\gamma \circ \mathrm{pr}_{B\mathrm{Spin}} \colon B^1(M) = K(\pi, 1) \times B\mathrm{Spin}$$

   where $\pi = \pi_1(M)$ and the map is projection to BSpin composed with the canonical fibration $\gamma \colon B\mathrm{Spin} \to BO$.

4. Recall that a manifold M is called stably *k-parallelisable* if for every k-dimensional complex K and every map $f\colon K \to M$, $f^*(TM \oplus \underline{\mathbb{R}})$ is trivial, where $TM \oplus \underline{\mathbb{R}}$ denotes the stable tangent bundle of M.

   Formulate a generalisation of the statements of the previous exercise for the normal j-type of a k-parallelisable manifold M with $j \leq k-1$.

   Hint: Recall that there are fibrations $\gamma_j\colon BO\langle j\rangle \to BO$ uniquely determined by the property that $\pi_i(BO\langle j\rangle) = 0$ if $i < j$ and $(\gamma_j)_*\colon \pi_i(BO\langle j\rangle) \cong \pi_i(BO)$ if $i \geq j$.

   Recall also that for every space X there is a $(j+1)$-equivalence $X \to P_j(X)$ where $P_j(X)$ is the j-th Postnikov stage of X.

5. As far as possible, determine the normal k-types, $k \leq [\dim(M)/2]$, for the manifolds M in the list from Lecture 1.

6. Record as much information as you can about the invariants of the manifolds listed in lecture 1: (co)homology groups and ring structure, homology groups, homotopy groups, tangential structure – e.g. are they orientable, are they spin.

7. Determine the following bordism groups in terms of the integral homology groups of the given Eilenberg-MacLane spaces. Groups marked with $*$ are particularly challenging:

   (a) $\Omega_4^{SO}(K(\pi,1))$,
   (b) $\Omega_5^{SO}(K(\pi,1))$,
   (c) $\Omega_4^{\mathrm{Spin}}(K(\pi,1))$ $*$,
   (d) $\Omega_5^{\mathrm{Spin}}(K(G,2))$, G a finite abelian group $*$,
   (e) $\Omega_6^{\mathrm{Spin}}(K(\mathbb{Z}^r,2))$, $r \geq 1$ $*$,
   (f) $\Omega_7^{\mathrm{Spin}}(K(\mathbb{Z}^r,2))$, $r \geq 1$ $*$.

8. Complete the following theorem due to Thom and Milnor and Wall:

   Theorem 1.1. *Closed oriented n-manifolds M_0 and M_1 are oriented bordant if and only if* ... .

9. List the bordism groups Ω_*^B for $0 \leq * \leq k$ for $k \leq 16$ as large as you can achieve and for the types B below. If possible give the invariants which detect the bordism class in the dimension $*$ and the type B:

   (a) $B = BO$
   (b) $B = BSO = BO\langle 2\rangle$
   (c) $B = B\mathrm{Spin} = BO\langle 4\rangle$
   (d) $B = B\mathrm{String} = BO\langle 8\rangle$

2 Tuesday

Exercises 5, 6 and 7 are about symmetric bilinear forms over the integers. Please see the paragraph below for the definitions of some of the terms used in these exercises.

1. Determine whether or not the following pairs of 4-manifolds are stably diffeomorphic or strictly stably diffeomorphic:

   (a) $4\mathbb{C}P^2\sharp(-2\mathbb{C}P^2)$ and $2\mathbb{C}P^2\sharp_2(S^2\times S^2)$,

   (b) $(-K3)\sharp\mathbb{C}P^2$ and $17\mathbb{C}P^2$: $-K3$ denotes a $K3$-surface with reversed orientation.

   (c) $(-K3)\sharp(-\mathbb{C}P^2)$ and $15\mathbb{C}P^2\sharp_4(S^2\times S^2)$.

2. (a) Let M_0 and M_1 be closed smooth 2-connected 6-manifolds. Prove that M_0 and M_1 are strictly stably diffeomorphic if and only if they have equal Euler characteristic: $\chi(M_0)=\chi(M_1)$

   (b) For each non-negative integer r find a 6-manifold as above M with $\chi(M)=2-2r$.

   (c) For 6-manifolds as above, do you think that stable diffeomorphism implies diffeomorphism?

3. Determine the normal 2-type of the following closed smooth simply connected 7-manifold:

$$M_{i,j} := SU(3)/S^1$$

   where for coprime integers i and j we embed $S^1\to SU(3)$ by the homomorphism

$$\lambda \mapsto \begin{bmatrix} \lambda^i & 0 & 0 \\ 0 & \lambda^j & 0 \\ 0 & 0 & \lambda^{-(i+j)} \end{bmatrix}.$$

4. Recall that $\mathrm{Spin}^{\mathbb{C}}(n)$ is the Lie group which sits into a short exact sequence

$$1\to S^1\to \mathrm{Spin}^{\mathbb{C}}(n)\to SO(n)\to 1$$

   which is the principal S^1 defined by the double covering $\mathrm{Spin}(n)\to SO(n)$. In particular $\mathrm{Spin}(n)\subset\mathrm{Spin}^{\mathbb{C}}(n)$ and there is a fibre sequence $BS^1\to B\mathrm{Spin}^{\mathbb{C}}(n)\to BSO(n)$ (and in the limit a fibre sequence $BS^1\to B\mathrm{Spin}^{\mathbb{C}}\to BSO$.)

   (a) Prove that an orientable M admits a normal $\mathrm{Spin}^{\mathbb{C}}$ structure if and only if $w_2(M)\in H^2(M;\mathbb{Z}/2)$, the second Stiefel-Whitney class of M, admits an integral reduction $W_2\in H^2(M;\mathbb{Z})$.

   (b) Prove that every orientable closed 4-manifold M admits a normal $\mathrm{Spin}^{\mathbb{C}}$ structure.

   Hint: This is very difficult! As a special case, first assume that $\pi_1(M)$ is an finite group of odd order.

5. Complete the following well known theorem from classical linear algebra

   Theorem 2.1. *Two indefinite unimodular symmetric bilinear forms over $\mathbb{Z}$ are isometric if and only if ... .*

6. Determine whether the following forms are stably equivalent:

$$\lambda_0 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \quad \text{and} \quad \lambda_0 \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

   Hint: You will need to consider the "*linking form*" that these non-degenerate forms induce on their "boundaries".

7. (a) Find a positive definite even unimodular form with non-zero signature.
   (b) Find positive definite even unimodular forms λ_0 and λ_1 with the same signature but which are not isometric.

   Hint: Here "find" means "find a reference" or "ask someone". If you try this on your own, expect a very challenging problem!

8. Let $K \in S^3$ be a knot. Recall that a framing of K is an embedding $\phi\colon D^2 \times S^1 \to S^3$ with $\phi(\{0\} \times S^1) = K$ and that surgery on ϕ is the closed 3-manifold

$$\chi(\phi) := \big(S^3 - \phi(\mathrm{int}(D^2 \times S^1)\big) \cup (S^1 \times D^2).$$

   Equivalence classes of framings are parametrised by $\pi_1(SO(2)) \cong \mathbb{Z}$ and these equivalence classes are detected by the *linking number* of the oriented knot, $\phi(\{0\} \times S^1)$, and the longitude $\phi(\{\epsilon\} \times S^1)$ for $0 < \epsilon \leq 1$. Here the linking number of a pair embedded oriented circles in S^1 is by the homology class of the first circle defines in the complement the second.

   (a) Verify that ± 1 surgery on a knot K is an integral homology 3 sphere.
   (b) If ϕ has linking number k, define the smooth 4-manifold

$$W(K, k) := D^4 \cup_{S^3} \omega(\phi)$$

   where $\omega(\phi)$ is the *trace* of the surgery on ϕ. Verify that $W(K, k)$ has the homotopy type $W(K, k) \simeq S^2$ and that the intersection form of $W(K, k)$ is $[\pm k]$.

9. Generalise the above exercise to links $L \subset S^3$ where for an r component link there is a diffeomorphism $L \cong \sqcup_{i=1}^{r} S^1$.

10. Find knots K_0 and K_1 such that the 4-manifolds $W(K_0, 1)$ and $W(K_1, 1)$ are not stably diffeomorphic.

Let $\lambda\colon H \times H \to \mathbb{Z}$ be a symmetric bilinear form over the $\mathbb{Z}$ where $H \cong \mathbb{Z}^r$ is a finitely generated free abelian group. A form b is called *even* if $\lambda(x, x) \in 2\mathbb{Z}$ for all $x \in H$ and *odd* otherwise. A form is *non-degenerate* if the adjoint homomoprhism $\hat{\lambda}\colon H \to H^*$ is injective and uni-modular if λ is an isomorphism: here $H^* = \mathrm{Hom}_{\mathbb{Z}}(H, \mathbb{Z})$ is the dual of H and $\hat{\lambda}(x)(y) = \lambda(x, y)$ for all $x, y \in H$. A form is called *positive definite* if for all $x \in H - \{0\}$, $\lambda(x, x) > 0$, *negative definite* if for all $x \in H - \{0\}$, $\lambda(x, x) < 0$ and *indefinite* otherwise.

The standard hyperbolic form is given by the matrix

$$H_+ := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Two forms λ_0 and λ_1 are called stably equivalent if there are integers r and s and an isometry

$$\lambda_0 \oplus_r H_+ \cong \lambda_1 \oplus_s H_+.$$

3 Thursday

1. Let M be a closed oriented smooth simply connected 4-manifold and let $L \in H^2(M;\mathbb{Z})$ be an integral lift of $w_2(M)$. Recall from Tuesday's final lecture the normal 2-type of M which is given by

$$B := B(w_2(M)) \cong K(H_2(M), 2) \times B\mathrm{Spin} \xrightarrow{L \times \gamma_4} BO \times BO \xrightarrow{\oplus} BO.$$

   If N is another closed oriented smooth simply connected 4-manifold prove that there is a ring isomorphism $H^*(M;\mathbb{Z}) \cong H^*(N;\mathbb{Z})$ if and only if there are bordant normal 2-smoothings $\bar{\nu}_M \colon M \to B$ and $\bar{\nu}_N \colon N \to B$ in the following cases

   (a) M is spinable, $w_2(M) = 0$,

   (b^*) M is not spinable, $w_2(M) \neq 0$.

   Hint: This last case is complicated by the fact that the differentials the AHSS of "twisted types" are more involved. E.g. in this case, d_2^1 is dual to the operation $Sq^2 \cup w_2$.

2. Show that $\mathbb{CP}^2 \# 2\overline{\mathbb{CP}^2}$ and $(S^2 \times S^2) \# \overline{\mathbb{CP}^2}$ are diffeomorphic 4-manifolds. (Observation: this shows that there is no 'unique prime factorization' for 4-manifolds.)

3. Let E_8 denote the negative definite even form with signature -8 and rank 8. Consider the forms $E_8 \oplus \langle -1 \rangle$ and $9\langle -1 \rangle$. Verify that these two forms are both odd, are of the same rank and signature but still inequivalent.

4. Suppose that D is a Kirby diagram representing the 4-manifold (with boundary) $X(D)$. Determine $\pi_1(X(D))$ and $H_*(X(D);\mathbb{Z})$.

5. (a) Let D be the diagram consisting of a single unknot with framing n. Show that $X(D)$ is the disk bundle over S^2 with Euler number n.

   (b) Let D_0 be the diagram consisting of a single unknot with framing 0. Determine the closed 4-manifold $Z(D)$.

   (c) Show a diagram D for which $Z(D)$ is $\mathbb{CP}^2$ and another one for which it is $\overline{\mathbb{CP}^2}$.

   (d) Let D be the Hopf link, with a dot on one component and framing n on the other. What is $X(D)$?

6. Let $M \to BO\langle 2k+1 \rangle$ be a $(2k-1)$-smoothing with source a compact connected $4k$-manifold. Prove the following:

   (a) If the boundary of M is a homotopy sphere, prove that the tangent bundle TM is trvial.

   (b) If M is closed let $c \colon M \to S^{4k}$ be the map collapsing the exterior of a small $4k$-disc in M to a point. Then there is a vector bundle ξ over S^{4k} such that the tangent bundle of M is isomorphic to the pull back of ξ along c: $TM \cong c^*\xi$.

   (c^*) Regarding $S\xi$, the stabilisation of ξ for part (b), as an element of $\pi_{4k-1}(SO)$, show that $J(S\xi) = 0 \in \pi_{4k-1}^S$.

7. Let M^4 be a closed smooth spin 4-manifold. Assuming Ex 6, prove Rohlin's theorem which states

$$\sigma(M^4) \in 16\mathbb{Z}.$$

8. Let $\Sigma \in \Theta_{4k-1}$ be a homotopy sphere. By Monday Exercise 4, Σ admits a normal smoothing in $BO\langle 2k{+}1\rangle$. For $k = 2, 3$ use Exercise 6 above, the main surgery theorem for $4k$-dimensional bordism and the facts below to determined the finite abelian groups Θ_7 and Θ_{11}.

9. Let M_0 and M_1 be a closed smooth oriented 4-connected 10-manifolds with equal Euler characteristic and assume [E] below.

   (a) Show that there is a homotopy sphere $\Sigma \in \Theta_{10}$ such that M_0 and $M_1 \sharp \Sigma$ are diffeomorphic.

   (b) With M as above and zero bordant in $\Omega_{10}^{BO\langle 6\rangle}$) and $\Sigma \in \Theta_{10} - \{[S^{10}]\}$ an exotic sphere, show that M and $M\sharp\Sigma$ are not diffeomorphic.

   (c) With M as above and $\Sigma \in \Theta_{10}$ a homotopy sphere assume part (b) and show that $M \cong M\sharp\Sigma$ if and only if $\Sigma \cong S^{10}$.

   (d) Give a list of all closed smooth oriented 4-connected 10-manifolds up to orientation preserving diffeomorphism (you may assume the exotic 10-spheres in your list).

   Hint: compare with Tuesday Ex 2.

10. Give examples of pairs of closed smooth manifolds manifolds M_0 and M_1 which satisfy the following (where possible give examples is dimension 4 and in a dimension greater than 4):

    (a) homeomorphic but not diffeomorphic,

    (b) stably diffeomophic but not diffeomorphic,

    (c) homeomorphic but not stably diffeomorphic,

Facts you may wish to use for some of the exercises.

A The stable J-homomorphism is a homomorphism $\pi_k(SO) \to \pi_k^S \cong \Omega_k^{\mathrm{fr}}$ with range the stable k-stem and which will be discussed in Friday's lectures: for $k = 3, 7, 11$ we have that J_3, J_7 and J_{11} are isomorphic to the following surjective homomorphisms

$$J_3 \cong \mathbb{Z} \to \mathbb{Z}/24, \quad J_7 \cong \mathbb{Z} \to \mathbb{Z}/240 \quad \text{and} \quad J_{11} \cong \mathbb{Z} \to \mathbb{Z}/504.$$

B Let $\mathrm{Vect}_{4k+1}(S^{4k}) \cong \pi_{4k-1}(SO) \cong \mathbb{Z}$ be the set of isomorphism classes of rank $(k+1)$-vector bundles over the $4k$-sphere and let $\xi_j \in \mathrm{Vect}_{4k+1}(S^{4k})$ correspond to $j \in \mathbb{Z}$ under the above isomorphism. Then a theorem due independently to Bott and Kervaire states that

$$p_k(\xi_j) = a_k(2k-1)!jx$$

where $x \in H^{4k}(S^{4k};\mathbb{Z})$ is a generator and $a_k = \frac{3-(-1)^k}{2}$.

C Hirzebruch's signature theorem states that signature of a closed smooth oriented $4k$-manifold M is computed by a rational polynomial in the Pontrjagin numbers of M:

$$\sigma(M) = \langle L(p_1(M), \ldots, p_k(M)), [M]\rangle.$$

$$L_0 = 1, \quad L_1 = \frac{p_1}{3}, \quad L_2 = \frac{7p_2 - p_1^2}{45}, \quad L_3 = \frac{62p_3 - 13p_2p_1 + 2p_1^3}{3^3 \cdot 5 \cdot 7}.$$

D There are isomorphisms $\Omega_7^{BO\langle 5\rangle} = 0$ and $\Omega_{11}^{BO\langle 7\rangle} = 0$.

E The natural homormophism $\Theta_{10} \to \Omega_{10}^{BO\langle 6\rangle} \cong \mathbb{Z}/6$ is an isomorphism.

4 Friday

1. Let $\mathcal{S}(M) = \{[f\colon N \simeq M]\}$ be the structure set of a closed manifold (as defined in Friday's second lecture) and let $\mathcal{E}(M)$ be the group of homotopy self-equivalences of M. Recall that $\mathcal{E}(M)$ acts on $\mathcal{S}(M)$ by post composition:

$$\mathcal{S}(M) \times \mathcal{E}(M) \to \mathcal{S}(M), \quad ([f\colon N \to M]), [g] \mapsto [g \circ f\colon N \to M].$$

   Show that the set $\mathcal{M}(M) := \mathcal{S}(M)/\mathcal{E}(M)$ is in bijection with the set of diffeomorphism classes of manifolds homotopy equivalent to M.

2. Let M be a closed smooth oriented n-manifold. The *inertia* group of M is defined to be the following subgroup of Θ_n:

$$I(M) := \{[\Sigma] \in \Theta_n | M \sharp \Sigma \cong M\}$$

   where $\cong$ denotes orientation preserving isomorphism.

   Recall that by the h-cobordism theorem every exotic sphere in dimension 6 and higher is a twisted double

$$\Sigma_f \cong D^n \cup_f D^n$$

   for some orientation preserving diffeomorphism $f\colon S^{n-1} \cong S^{n-1}$. (This is also true in dimension 5 since there are no exotic 5-spheres). Set $M^\bullet := M - \mathrm{int}(D^n)$ and identify $\partial M^\bullet = S^{n-1}$. Show that $[\Sigma_f] \in I(M)$ if and only if there is an orientation preserving diffeomorphism $F\colon M^\bullet \cong M^\bullet$ with $F|_{S^{n-1}} = f$.

   Hint: You may assume a theorem of Cerf which states that all orientation preserving embeddings $D^n \to M^n$ of the n-disc into an oriented n-manifold are ambient isotopic.

3. Let $\mathbb{O}P^2$ be the octonionic projective plane. It is a closed smooth 7-connected 16-manifold and $(\mathbb{O}P^2)^\bullet$ is diffeomorphic to the 8-disc bundle $D^8 \to W^{16} \to S^8$ associated to the Hopf fibration $S^7 \to S^{15} \to S^8$.

   (a) Show one or other of the following:

      i. The normal 7-type of $\mathbb{O}P^2$ is $\gamma_8\colon B\mathrm{String} \to BO$.

      ii. The normal 8-type of $\mathbb{O}P^2$ is $\gamma_8\colon B\mathrm{String} \to BO$. (You may use that fact that multiplication by $\eta \in \pi_1^s$ defines a surjection $\eta\colon \pi_8(B\mathrm{String}) \to \pi_9(B\mathrm{String})$).

   (b) Given that $\Omega_{16}^{\mathrm{String}} \cong \mathbb{Z}^2$ and that Θ_{16} is a finite group (in fact $\Theta_{16} \cong \mathbb{Z}/2$) show that $I(\mathbb{O}P^2) = \Theta_{16}$.

   Hint: The stabilisation of the clutching function of the Hopf fibrataion $S^7 \to S^{15} \to S^8$ generates $\pi_8(BO) \cong \pi_8(B\mathrm{Spin})$.

4. Let $\lambda : F \times F \to \mathbb{Z}$ be a uni-modular symmetric bilinear form on a finitely generated free abelian group F.

   (a) Assuming the following:

   If λ is indefinite then there exists $x \in F$ such that $x \neq 0$ and $\lambda(x, x) = 0$,

   prove that sign : $L_0(e) \to \mathbb{Z}$ is injective

   (b) An element $x_0 \in F$ is called *characteristic* for λ if $\lambda(x, x) \equiv \lambda(x, x_0)$ mod 2 for all $x \in F$. Prove the following:

i. Characteristic elements exist

ii. The quantity $\mathrm{char}(\lambda) := \lambda_0(x_0, x_0) \bmod 8$, is independent of the choice of x_0 and hence an invariant of the isometry type of λ.

iii. Prove that $\mathrm{char}(\lambda_0 \oplus \lambda_1) = \mathrm{char}(\lambda_0) + \mathrm{char}(\lambda_1) \in \mathbb{Z}/8$.

iv. The (symmetric) L-group $L^0(e)$ is defined defined to be the Grothendieck of unimodular symmetric bilinear forms over $\mathbb{Z}$ modulo the subgoup generated by forms which are metabolic where a symmetric form λ is called metabolic if it contains a half-rank summand $L \subset F$ on which λ vanishes identically: $\lambda(L, L) = 0$.
Assuming that the signature defines an isomorphism, $\sigma\colon L^0(e) \to \mathbb{Z}$ (see Krakow 2010 Friday, Algebra 2) use the above considerations to show that the signature of every even unimodular form is divisible by 8. Hence prove that the signature defines an isomorphism

$$\sigma\colon L_0(e) \to 8\mathbb{Z}.$$

Hint: There are not typos: there is an injective homomorphism $L_0(e) \to L^0(e)$.

5. Let (F, λ, μ) be a (-1)-symmetric unimodular quadratic form over $\mathbb{Z}$ so that $(F, \lambda) \cong H_-(\mathbb{Z}^r)$ with canonical basis $\{e_1, \dots, e_r, f_1, \dots, f_r\}$. Recall that $\mu\colon F \to \mathbb{Z}/2$ is a function such that for all $x, y \in F$,

$$\mu(x + y) = \mu(x) + \mu(y) + \lambda(x, y) \quad \mathrm{mod} 2.$$

Define the *Arf invariant* of (F, λ, μ) by the equation

$$A(F, \lambda, \mu) = \Sigma_{i=1}^{r} \mu(e_i)\mu(f_i) \in \mathbb{Z}/2.$$

Prove that the the Arf invariant is well-defined and indeed defines an isomorphism

$$A\colon L_2(e) \cong \mathbb{Z}/2.$$

Hint: Start by classifying quadratic forms on $\mathbb{Z}^2$, use induction and also count the size of the sets $\mu^{-1}(0)$ and $\mu^{-1}(1)$.

6. Let $a = \Pi_{i=1}^{j} p_i$ be a product of distinct odd primes and let $[2a] = (\mathbb{Z}, 2a)$ be the indicated rank one even symmetric bilinear form. Show that up to isometries of $H_+(\mathbb{Z})$, there are precisely 2^{j-1} embeddings of $[2a]$ into $H_+(\mathbb{Z})$.

Hint: The group of isometries of $H_+(\mathbb{Z})$ is isomorphic to $\mathbb{Z}/2 \oplus \mathbb{Z}/2$ with generators represented by

$$T := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad -\mathrm{Id} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

5 Organisation

5.1 Monday - Solvers

1.
2. Bela Racz :)
3. Andrzej Czarnecki :)
4. Johannes Nordstrom :)
5. Wojciech Politarczyk
6.
7. (a)
   (b)
   (c)
   (d)
   (e) Marek Kaluba
   (f) Marek Kaluba
8.
9. (a)
   (b)
   (c)
   (d)

5.2 Tuesday - Solvers

1. Inanc Baykur :)
2. Ahmet Beyaz :)
3.
4. Raphael Zentner :)
5. Khaled Qazaqzeh :)
6.
7. Rafael Torres
8.
9. Andrew Donald :)
10. Ju A Lee :)

5.3 Thursday - Solvers

1. Anna Abczynski
2. Antonio Rieser
3. Vera Vertesi and Raphael Zentner
4.
5.
6.
7.
8.
9. Anna Abczynski :)
10.

5.4 Friday - Solvers

1. Daniel Kasprowski :)
2. Antonio Rieser
3.
4.
5.
6.